Advanced Higher Maths – Success Criteria

Skills, knowledge and understanding for the course:

- ✓ Use mathematical reasoning skills to think logically, provide justification, and solve problems.
- ✓ Gain knowledge and understanding of a range of complex concepts.
- ✓ Select and apply complex operational skills
- ✓ Use reasoning skills to interpret information and complex mathematical models.
- ✓ Effectively communicate solutions in a variety of contexts.
- ✓ Explain and justify concepts through the idea of rigorous proof.
- ✓ Think creatively.

Skills, knowledge and understanding for the assessment:

Topic	I can…
Partial Fractions	☐ Decompose a proper rational function as a sum of partial fractions where the denominator may contain distinct linear factors, an irreducible quadratic factor, or a repeated linear factor ☐ Reduce an improper rational function to a polynomial and a proper rational function by division or otherwise
Binomial Theorem	☐ Use the binomial theorem to expand an expression of the form $(ax^p + by^q)^n$ ☐ Use the general term for a binomial expansion, finding a specific term in an expression
Differential Calculus	☐ Differentiate functions involving $\ln(x)$ and $e(x)$ ☐ Apply the chain rule to differentiate the composition of at most three functions ☐ Differentiate functions of the form $f(x)g(x)$ and $\frac{f(x)}{g(x)}$ ☐ Recall the definitions and can apply the derivatives of $\tan(x)$, $\cot(x)$, $\sec(x)$ and $cosec(x)$ ☐ Derive and use the derivatives of $\tan(x)$, $\cot(x)$, $\sec(x)$ and $cosec(x)$ ☐ Differentiate functions that require more than one application or combination of applications of chain rule, product rule, and quotient rule ☐ Apply $\frac{dy}{dx} = \frac{1}{(\frac{dx}{dy})}$ where appropriate ☐ Differentiate inverse trigonometric functions ☐ Use differentiation to find the first derivative of a relationship defined implicitly, including in context ☐ Use differentiation to find the second derivative of a relationship defined implicitly ☐ Use logarithmic differentiation; recognising when it is appropriate in extended products, quotients, and in functions where the variable occurs in an index ☐ Apply differentiation to related rates in problems where the relationships may or may not be given ☐ Use differentiation to find the first derivative of a relationship defined parametrically ☐ Apply parametric differentiation to motion in a plane, including instantaneous speed ☐ Use differentiation to find the second derivative of a relationship defined parametrically

	Apply differentiation to problems in context Apply differentiation to optimisation
Systems of Linear Equations	Find the solution to a system of equations Ax = b, where A is a 3 × 3 matrix and where the solution is unique — candidates should understand the term 'augmented matrix' Show that a system of equations has no solutions (inconsistency) Show that a system of equations has an infinite number of solutions (redundancy) Compare the solutions of related systems of two equations in two unknowns and recognise ill-conditioning
Properties of Functions	Find the vertical asymptote(s) to the graph of a rational function Find the non-vertical asymptote to the graph of a rational function Investigate features of graphs • points of inflection • stationary points • domain and range • odd, even or neither • continuous or discontinuous • extrema of functions: the maximum and minimum values of a continuous function f defined on a closed interval [*a,b*] can occur at stationary points, end points, or points where f' is not defined Sketch graphs using features given or obtained Sketch related functions: • modulus functions • inverse functions • functions differentiated • translations and reflections
Complex Numbers	Plot complex numbers in the complex plane (an Argand diagram) Recall the definition of modulus and argument of a complex number Convert a given complex number from Cartesian to polar form and vice-versa Use de Moivre's theorem with integer and fractional indices Apply de Moivre's theorem to multiple angle trigonometric formulae Apply de Moivre's theorem to find the nth roots of a complex number Interpret geometrically certain equations or inequalities in the complex plane by sketching or describing a straight line or circle that represents the locus of points that satisfy a given equation or inequality Perform the operations of addition, subtraction, multiplication, and division Find the square root Find the roots of a cubic or quartic equation with real coefficients when one complex root is given Solve equations involving complex numbers
ASSESSMENT 1	
Integral Calculus	Integrate $\cot(x)$, $\sec(x)$ and $cosec(x)$ Recognise and integrate product and quotient expressions Use partial fractions to integrate proper or improper rational functions Integrating by substitution Integrate where the substitution is given Integrating by parts Use integration by parts with one or more applications Apply integration to problems in context
Applications of Calculus	Apply integration to volumes of revolution, where the volume generated is by the rotation of the area under a single curve about the x-axis or y-axis

	□ Apply integration to the evaluation of areas, including integration with respect to y □ Apply integration to problems in context
Sigma Notation and Power Series	□ Apply the rules of sequences and series to find: • the nth term • the sum to n terms • common difference of arithmetic sequences • common ratio of geometric sequences □ Determine the sum to infinity of geometric series □ Determine the condition for a geometric series to converge □ Apply summation formulae □ Recall and use sums of certain series, and other straightforward results and combinations of these □ Use the Maclaurin expansion to find specified terms of the power series for simple functions □ Use the Maclaurin expansion to find a power series for simple functions □ Combine Maclaurin expansions to find a power series
FAB 1 ASSESSMENT	
Differential Equations	□ Find general and particular solutions to equations that can be written in the form $\frac{dy}{dx} + P(x)y = f(x)$ □ Find general and particular solutions of second order linear ordinary differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ (homogeneous) $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ (non-homogeneous) where the roots of the auxiliary equation may be: • real and distinct • real and equal • complex conjugates
Number Theory	□ Use Euclid's algorithm to find the greatest common divisor of two positive integers, for example using the division algorithm repeatedly □ Express the greatest common divisor (of two positive integers) as a linear combination of the two □ Express integers in bases other than 10 □ Recall and use the fundamental theorem of arithmetic
Formal Proofs	□ Disprove a conjecture by providing a counterexample □ Recall and use the symbols ∃ (there exists) and ∀ (for all) □ Give the negation of a statement □ Use indirect or direct proof in straightforward examples □ Prove a statement by contradiction □ Use proof by contrapositive □ Use direct proof in straightforward examples □ Use proof by induction
Matrices	□ Find the determinant of a 2 × 2 matrix and a 3 × 3 matrix □ Determine whether a matrix is singular □ Recall and apply $\det(AB) = \det(A)\det(B)$ □ Find the inverse of a matrix □ Recall and use the inverse of a 2 × 2 matrix □ Find the inverse of a 3 × 3 matrix □ Use transformation matrices

	Use 2 × 2 matrices to carry out geometric transformations in the plane — the transformations should include rotations, reflections, and dilatations Apply combinations of transformations Perform matrix operations (at most order three): addition, subtraction, multiplication by a scalar, multiplication of matrices Recall and apply the properties of matrix addition and multiplication: Recall and apply key properties of the transpose, the identity matrix, and inverse
Vectors	Use a vector product method in three dimensions Evaluate the scalar triple product **a** . (**b** x **c**) Work with lines in three dimensions Find the equation of a line in parametric, symmetric, or vector form, given suitable defining information Find the angle between two lines in three dimensions Determine whether or not two lines intersect and, where possible, finding the point of intersection Find the equation of a plane in vector, parametric, or Cartesian form Find the point of intersection of a plane with a line that is not parallel to the plane Determine the intersection of two or three planes Find the angle between a line and a plane, or between two planes
FAB 2 ASSESSMENT	

What will be taken into consideration when deciding on a teacher-estimated grade for Advanced Higher Mathematics?

- Internal Assessment 1 assessing 40% of the course.
- FAB 1 Assessment (Dec) assessing 65% of the course.
- FAB 2 Assessment (Feb/March) assessing 100% of the course.
- Commitment and Quality in class/homework.
- Attendance at Supported Study.